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ANNUITY STUDIES

SAMUEL F. RACINE

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Annuity Studies

—BY—

Samuel F. Racine

Certified Public Accountant

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The Accumulation of $1.00 at Compound Interest

Periods	1%	1¼%	1½%	2%	2¼%	2½
0	1.	1.	1.	1.	1.	1.
1	1.01	1.0125	1.015	1.02	1.0225	1.025
2	1.0201	1.02515625	1.030225	1.0404	1.04550625	1.050625
3	1.030301	1.03797070	1.04567838	1.061208	1.06903014	1.07689063
4	1.04060401	1.05094534	1.06136355	1.08243216	1.09308332	1.10381289
5	1.05101005	1.06408215	1.07728400	1.10408080	1.11767769	1.13140821
6	1.06152015	1.07738318	1.09344326	1.12616242	1.14282544	1.15969342
7	1.07213535	1.09085047	1.10984491	1.14868567	1.16853901	1.18868575
8	1.08285671	1.10448610	1.12649259	1.17165938	1.19483114	1.21840290
9	1.09368527	1.11829218	1.14338998	1.19509257	1.22171484	1.24886297
10	1.10462213	1.13227083	1.16054083	1.21899442	1.24920343	1.28008454
11	1.11566835	1.14642422	1.17794894	1,24337431	1.27731050	1.31208666
12	1.12682503	1.16075452	1.19561817	1.26824179	1.30604999	1.34488882
13	1.13809328	1.17526395	1.21355244	1.29360663	1.33543611	1.37851104
14	1.14947421	1.18995475	1.23175573	1.31947876	1.36548343	1.41297382
15	1.16096896	1.20482918	1.25023207	1.34586834	1.39620680	1.44829817
16	1.17257864	1.21988955	1.26898555	1.37278571	1.42762146	1.48450562
17	1.18430443	1.23513817	1.28802033	1.40024142	1.45974294	1.52161826
18	1.19614748	1.25057739	1.30734064	1.42824625	1.49258716	1.55965872
19	1.20810895	1.26620961	1.32695075	1.45681117	1.52617037	1.59865019
20	1.22019004	1.28203723	1.34685501	1.48594740	1.56050920	1.63861644
21	1.23239194	1.29806270	1.36705783	1.51566634	1.59562066	1.67958185
22	1.24471586	1.31428848	1.38756370	1.54597967	1.63152212	1.72157140
23	1.25716302	1.33071709	1.40837715	1.57689926	1.66823137	1.76461068
24	1.26973465	1.34735105	1.42950281	1.60843725	1.70576658	1.80872595
25	1.28243200	1.36419294	1.45094535	1.64060599	1.74414632	1.85394410
26	1.29525631	1.38124535	1.47270953	1.67341811	1.78338962	1.90029270
27	1.30820888	1.39851092	1.49480018	1.70688648	1.82351588	1.94780002
28	1.32129097	1.41599230	1.51722218	1.74102421	1.86454499	1.99649502
29	1.33450388	1.43369221	1.53998051	1.77584469	1.90649725	2.04640739

Periods	3%	3½%	4%	4½%	5%	6%
0	1.	1.	1.	1.	1.	1.
1	1.03	1,035	1.04	1.045	1.05	1.06
1	1.0609	1.071225	1.0816	1.092025	1.1025	1.1236
3	1.092727	1.10871788	1.124864	1.14116613	1.157625	1.191016
4	1.12550881	1.14752300	1.16985856	1.19251860	1.21550625	1.26247696
5	1.15927407	1.18768631	1.21665290	1.24618194	1.27628156	1.33822558
6	1.19405230	1.22925533	1.26531902	1.30226012	1.34009564	1.41851911
7	1.22987387	1.27227926	1.31593178	1.36086183	1.40710042	1.50363026
8	1.26677008	1.31680904	1.36856905	1.42210061	1.47745544	1.59384807
9	1.30477318	1.36289735	1.42331181	1.48609514	1.55132822	1.68947896
10	1.34391638	1.41059876	1.48024428	1.55296942	1.62889463	1.79084770
11	1.38423387	1.45996972	1.53945406	1.62285305	1.71033936	1.89829856
12	1.42576089	1.51106866	1.60103222	1.69588143	1.79585633	2.01219647
13	1.46853371	1.56395606	1.66507351	1.77219610	1.88564914	2.13292826
14	1.51258972	1.61869452	1.73167645	1.85194492	1.97993160	2.26090396
15	1.55796742	1.67534883	1.80094351	1.93528244	2.07892818	2.39655819
16	1.60470644	1.73398604	1.87298125	2.02237015	2.18287459	2.54035168
17	1.65284763	1.79467555	1.94790050	2.11337681	2.29201832	2.69277279
18	1.70243306	1.85748920	2.02581652	2.20847877	2.40661923	2.85433915
19	1.75350605	1.92250132	2.10684918	2.30786031	2.52695020	3.02559950
20	1.80611123	1.98978886	2.19112314	2.41171402	2.65329771	3.20713547
21	1.86029457	2.05943147	2.27876807	2.52024116	2.78596259	3.39956360
22	1.91610341	2.13151158	2.36991879	2.63365201	2.92526072	3.60353742
23	1.97358651	2.20611448	2.46471554	2.75216635	3.07152376	3,81974966
24	2.03279411	2.28332849	2.56330416	2.87601383	3.22509994	4.04893464
25	2.09377793	2.36324498	2.66583633	3.00543446	3.38635494	4.29187072
26	2.15659127	2.44595856	2.77246978	3.14067901	3.55567269	4.54938296
27	2.22128901	2.53156711	2.88336858	3.28200956	3.73345632	4.82234594
28	2.28792768	2.62017196	2.99870332	3.42969999	3.92012914	5.11168670
29	2.35656551	2.71187798	3.11865145	3.58403649	4.11613560	5.41838790

Annuity Studies

CHAPTER I.

Rules, Formulas and Typical Problems

A. ACCUMULATION OF AN AMOUNT

Rule:

The accumulation of $1.00 at interest is found by multiplying one plus the rate by itself as many times, less one, as there are periods.

Formula:

$$(1+r)^n$$

Typical Problem:

Required the accumulation of $200.00 in 10 years @ 4%.

B. COMPOUND INTEREST

Rule:

The compound interest on $1.00 is found by multiplying one plus the rate by itself as many times, less one, as there are periods, and then subtracting the principal, one.

Formula:

$$(1+r)^n-1$$

Typical Problem:

Required the compound interest on $500.00 for 4 years @ 6%.

C. PRESENT WORTH OF AN AMOUNT

Rule:

The present worth of $1.00 is found by dividing one by the accumulation of $1.00 at compound interest (A).

Formula:

$$\frac{1}{(1+r)^n}$$

Typical Problem:

Required the present worth of $1,000.00 due 4 years hence, money being worth 5%.

D. DISCOUNT ON AN AMOUNT

Rule:

The discount is found by subtracting the present worth (C) from the principal.

Formula:

$$1 - \frac{1}{(1+r)^n}$$

Typical Problem:

Required the discount on $500.00 due in 3 years at 5%.

E. ACCUMULATION OF AN ANNUITY

Rule:

The accumulation of an annuity of $1.00 is found by dividing the compound interest (B) by the rate of nominal interest.

Formula:

$$\frac{(1+r)^n - 1}{r}$$

Typical Problem:

What will be the accumulation of $200.00 per year in 4 years at 3%.

F. PRESENT WORTH OF AN ANNUITY

Rule:

The present worth of an annuity is found by dividing the discount (D) by the rate of nominal interest.

Formula:

$$\frac{1-\frac{1}{(1+r)^n}}{r}$$

Typical Problem:

Required the present worth of an annuity of $200.00 per year with 4 years to run at 3%.

G. AMOUNT OF AN ANNUITY (SINKING FUND)

Rule:

The amount of an annuity, or the sinking fund, which will produce one dollar is found by dividing the nominal interest by the compound interest (B).

Formula:

$$\frac{r}{(1+r)^n-1}$$

Typical Problem:

Required the amount of a sinking fund which will produce $540.00 in 3 years at 6%.

H. RENT OF AN ANNUITY. (The amount which, when periodically applied to an interest bearing principal, will amortize it in a given time.)

Rule:

To find the rent of an annuity of $1.00, divide the rate by the discount (D).

Formula:

$$\frac{r}{1 - \frac{1}{(1+r)^n}}$$

Typical Problem:

"A" owes $10,000.00 with interest at 6%. What uniform amount should he pay annually for 5 years to cover the obligation?

I. PREMIUM OR DISCOUNT

Rule:

To find the premium or discount on a $1.00 bond or similar obligation:

(a) Divide the difference between the nominal and effective rates of interest by the effective rate and multiply by the discount (D).

Formula:

Premium:

$$\frac{nr - er}{er}\left(1 - \frac{1}{(1+er)^n}\right)$$

Discount:

$$\frac{er - nr}{er}\left(1 - \frac{1}{(1+er)^n}\right)$$

Typical Problem:

Premium: A 5% bond, with 2 years to run, is sold to net the investor 4½%. What is the premium?

Discount: A 4½% bond, with 4 years to run, is sold to net the investor 5%. What is the discount?

J. RESIDUAL VALUE

Rule:

The amount to which a given amount may be reduced in a given number of periods at a given rate is found by subtracting the rate from one; raising the remainder to the power equivalent to the number of periods, and multiplying by the given amount.

Formula:

$$A(1-r)^n$$

Typical Problem:

What is the residual value of a machine costing $1200.00 depreciated at 10% on reducing balances for 5 years?

K. RATE OF REDUCTION, OR DEPRECIATION

Rule:

The rate which will reduce a given amount to a lesser amount (residual) in a given number of periods is found by dividing the residual by the given amount and extracting the root equivalent to the number of periods, then subtracting this amount from one.

Formula:

$$1-\sqrt[n]{\frac{R}{A}}$$

Typical Problem:

An automobile costing $2,000.00 has a life of 3 years and a residual value of $1,024.00. What is the rate of depreciation on decreasing balances?

CHAPTER II.

Illustrative Problems.

GROUP I.

1. If money is worth 6%, what will $3,000 amount to in 4 years if the interest is collected semi-annually and re-invested promptly?

2. What is the present worth of $30,000 payable in 4 years if money is worth 6%?

3. If you receive an annuity of $10,000 per year, payable annually at the end of each year, how much would you have at the end of 3 years provided, however, that the 3rd payment had not been received and that you had re-invested the money as soon as received at 5% annually?

4. If money is worth 6%, what is the value of an annuity of $2,000 with 4 years to run?

5. What is the present worth of $18,000 due in 5 years at 4%?

GROUP II.

1. What is the residual value of a machine costing $4,000.00 in 5 years at 10%?

2. What is the residual value of a machine costing $3,000.00 in 4 years at 15%?

3. The Smith Manufacturing Company acquires an automobile for $1,500.00. They anticipate trading it in on another machine in 2 years at a value of $486.00. What is the rate of depreciation on decreasing balances?

4. Suppose the cost was $4,000.00 and the residual value $1,687.50.

5. Which is the better purchase, and what is the saving during each of the first 3 years, on a machine costing $2,000.00 with a 4 years life and a residual value of $819.20, over a machine costing $1,800.00 having a 3 years life with a residual value of $1029.22.

GROUP III.

1. Jno. Smith has an obligation of $19,500.20 due 2 years hence. What amount should he invest at intervals of six months in order to accumulate the desired amount if he is able to secure 5% interest on his investment?

2. Jones is entitled to an annuity of $50,000 per year for two years payable in quarterly installments. How much would you be willing to pay for this annuity if money were worth 5% and the next installment were due 3 months from today?

3. An arrangement is made whereby $15,000 is deposited every six months in a trust company at 3% interest. Provided that these deposits are continued for 8 periods, what would be the amount of the sinking fund?

4. What is the present worth of $1,000 due in 6 years at 6%?

5. What is the present worth of the interest which would be received on five $1,000 bonds bearing 6% interest, due in 4 years if money is worth 6%?

GROUP IV.

1. The John Doe Investment Company contemplates loaning approximately $20,000 to the Appleby Manufacturing Co. and receiving therefrom $1,000, 5% gold bonds. They desire a return of 6% on their money; what is the exact amount they will loan which will nearest equal $20,000 and how many bonds will they receive? Time, 10 years.

2. Which, if either of the following amounts correctly represent the value of ten 4½% bonds due in 3 years on a 4% basis, interest payable semi-annually? A, $10,138.75; B, $9,859.96.

3. (a) What premium should be paid on five $1,000 bonds maturing in 3 years with interest at 5% payable semi-annually on a 4½% basis?

(b) How much would be the discount on a 6% basis?

(c) Suppose that the bonds bore 4% interest, payable semi-annually, what would be the amount of the discount?

4. You are offered five $1,000 5% bonds payable in 3 years at 101⅜ and five $1,000 4½% bonds payable in 3 years at 101½. Which is the better purchase, presuming that 4½% on the first group of bonds and 4% on the second group of bonds are fair rates of return considering the nature of the investment?

5. Prepare a table showing the amortization of a 6% bond on a 5% basis for 4 years.

GROUP V.

1. Bonds due in 2 years bearing 6% interest, payable semi-annually, and yielding 2½% each half year are worth 1.01881. Prepare an amortization table showing the value in this bond for each half year of the two years.

2. Required the ledger accounts in detail to record the following transactions:

> 10, 4% bonds are purchased on a given date, just after the interest coupons for that date had been removed, at 87⅜. Nine months later the bonds were sold for 88½ and interest. What was the profit?

3. (a) $5,000 of 4½% bonds, interest payable semi-annually, with two years to run, were purchased for $4,850. The purchaser estimated that the bonds would net him 3% per half year. To what amount did he err in his estimate?

(b) Three months later he is offered 97½ and interest for his bonds. Presuming that he sells the bonds, what entries will be required on his books.

4. A piece of real property is being sold, subject, however, to the rights of a life tenant. The remainderman estimates that the property is worth $10,000. Presuming that the life tenant's equity runs for 6 years and that the property produces 6% on the estimated value, what amount should be paid to the life tenant to satisfy both the life tenant and the remainderman?

5. Given a 15 year annuity of $60.00, the first payment of which falls due one year hence.

(a) What is the value of the annuity at 5%?

(b) What amount will accumulate during the period if each moiety is reinvested as it becomes due.

Assume interest at the rate of 5%, payable annually.

6. What amount will be required to produce $20,000 in 10 years, provided $2,000 per year has been set aside for three of the 10 years and that these amounts, with the future annual payments of the remaining 7 years, bear 4% interest?

CHAPTER III.

General Problems.

1. Required the present worth of an annuity of $50.00, the first payment of which falls due one year hence and continues for 25 years. Interest 5%.

2. If each moiety of the preceding question were invested promptly, what would be the accumulation at the end of the period?

3. You have been requested to advise the amount of a sinking fund which will produce $500,000.00 in 25 years, amounts to be invested annually at 5%. What is the amount?

4. A corporation has outstanding an issue of 20-year 6% bonds, with 10 years to run, which were sold to net the investor 5%. They now have an opportunity to buy $100,000.00 of these bonds at 102. Is it advisable for them to buy? If so, what will they save, also what entries should be made on their books of account to record the purchase?

5. Under the terms of an agreement, a debt of $100,000.00 with interest at 5% is to be paid as follows: Nine equal annual payments and a tenth payment of $12,000.00. What is the amount of the annual payments?

6. What should be paid for a 6% bond, interest payable semi-annually with 3 years to run, if it is to net 5%?

7. What is the present worth of an annuity of $700.00 for 7 years at 5%?

8. Find the annuity whose amount for 25 years at 6% is $16,459.35.

9. A man bought a tract of land for $4,800.00 which was to be paid in installments of $600.00 per year. How much money at 6% interest (compound) would discharge the debt at the time of purchase?

10. Required the present worth of the following annuities:

	Amount	Periods	Rate
A.	$500.00	12	2%
B.	800.00	6	4%
C.	500.00	11	1½%

11. Ascertain the accumulation of the amounts in question 10 at the rates and for the periods given.

12. What is the rent of an annuity of 15 periods, if the present worth is $1,200.00 and the rate 1½%?

13. What amount, set aside semi-annually, will produce $1,000.00 in 12 years at 4% interest?

14. If interest at 6% per annum is paid monthly, what is the effective rate?

15. If a person receives $15.00 per quarter on $1,000.00, what is the effective rate of interest per annum?

16. Which is the more valuable and how much: $8,160.00 payable annually or $2,000.00 each quarter, interest 5%?

17. If money is worth 5% per annum, what rate should be quoted where payments are to be made quarterly?

18. If a 4% bond nets 2½% semi-annually what is its value —5 years to run?

19. As above, but 3% netting 2½% semi-annually—7 years to run?

20. Same as above, but 5% netting 6% annually—12 years to run.

21. Same as No. 20 but netting 3½% semi-annually?

22. Prepare a table showing the annual book values and the amortization of a 6% bond netting 2½% semi-annually—9 years to run.

23. On March 1st, 1918, $30,000.00 3% bonds, due July 1st, 1920, J. & J., were sold, netting 4%. What is the price flat?

24. What are the intermediate payments on an obligation of $200,000.00 bearing 6% interest, payable $20,000.00 cash: 9 equal annual payments and a final payment in 10 years of $8,000.00.

25. If, in question 24, the final payment had been $28,000.00, what would have been the amount of the 9 intermediate payments?

26. What will be the residual value of a machine costing $5,000.00 with an estimated life of 12 years, depreciated at 15% annually?

27. What rate should be used in problem No. 26 to produce a residual value of $1,315.00 in 10 years?

28. What is the bid on $100,000.00 5% bonds maturing at the end of 3 years, interest payable semi-annually, to net the purchaser a nominal rate of 4%?

29. What is the bid on $100,000.00 3% bonds maturing at the end of 6 years, interest payable semi-annually, to net the purchaser a nominal rate of 4%?

30. On August 1st, 1919, $5,000.00 5% bonds, due April 1st, 1921, are offered at prices to yield 4½%. What is the price "and interest"?

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